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OPERATIONS EVALUATION GROUP

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INTERIM RESEARCH MEMORANDUM

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COMBAT EFFECTIVENESS OF ALLIED AND GERMAN TROOPS IN THE WORLD WAR II INVASION OF CRETE

By

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ABSTRACT

An analysis is made of statistics pertaining to the landing of German troops, and landing or withdrawal of British and allied troops during the invasion of Crete by the Germans in World War II, using information on total casualties experience on each side, and assumption that Lanchester's Law holds.

The combat effectiveness of the average individual troops on each side are computed and compared with similar figures for the battle of Iwo Jima.

1. An examination of the data of reference (a), plus a use of a small number of assumptions has made it possible to provide an estimate of the combat effectiveness of the forces which took part in the battle of Crete. The combat effectiveness thus obtained may be compared with the combat effectiveness obtained for the forces which took part in the battle of Iwo Jima, as derived in reference (b). This analysis is based on work originally produced in internal OEG documents in 1954.

2. The basic assumption for this analysis is that:

(A) The casualty-producing rate of an entire force is equal to the number of troops in the force multiplied by the casualty-producing rate of the average combatant in the force. This assumption leads to results commonly known as the Lanchester Square Law.

Insufficient data is presented in reference (a) to verify whether or not assumption (A) is valid for the invasion of Crete. Whether such data can be found in the references cited in reference (a) has not been determined. Other assumptions which have been made pertain to the rates at which the German invaders put their troops ashore, the rates at which the combined British and Greek defenders reinforced and evacuated their troops, and the total numbers of casualties inflicted on both sides. These assumptions have been derived from more or less fragmentary information found in various portions of reference (a). These assumptions may be listed as follows:

(B) 8,100 German troops were landed the first day, 7,400 the second day, and a total of 9,500 more uniformly during the third through ninth days.

(C) 27,550 British and 13,000 Greeks, for a total of 40,550, were on hand at the beginning of the battle. Included in the total British troops on hand were an unarmed labor force of 4,000-5,000 Cyprians and Palestrinians and an unspecified number of sick, wounded and war-weary troops all of whom were counted initially as effective troops for purposes of this analysis. A total of 950 additional British troops were landed on the eighth day. Two Greek battalions took to the hills (2,000 men) when their ammunition ran out early during the battle. These 2,000 departures were distributed uniformly over the third through fifth days of the battle. Other Greeks who were not listed as killed or captured, 2,800 in all, were counted as departures distributed uniformly through the sixth through thirteenth days. 4,000 British troops were evacuated on the tenth day, an additional 11,000 were evacuated uniformly distributed through the tenth through thirteenth days, and an additional 1,000 were evacuated on the thirteenth day.

(D) The German estimate of their own casualties was about 6,000. The British estimate of the German casualties was 9,000 German wounded, 6,000 German killed.

(E) There were 2,600 British killed and 2,600 Greeks killed for a total of 5,200 killed. There were 10,500 British prisoners and 5,600 Greek prisoners taken for a total of 16,100 prisoners. This gave a total of 21,300 casualties in all. Wounded who were successfully evacuated, and troops who died while being evacuated have not been counted as British or Greek casualties.

TABLE I

ESTIMATE OF TROOP LANDINGS AND WITHDRAWALS ON CRETE

Time	German troops (M) $M(0) = 0$		British troops N_1 $N_1(0) = 27,550$	Greek troops N_2 $N_2(0) = 13,000$	Combined Allied troops (N) $N_1(0)+N_2(0) = N(0) = 40,550$
(t, t+1)	p(t, t+1)	$M(0) + \sum_{i=0}^{t} p_M(i, i+1)$	$P_{N_1}(t, t+1)$	$P_{N_2}(t, t+1)$	$N(0) + \sum_{i=0}^{t} P_N(i, i+1)$
0- 1	8,100	8,100	0	0	40,550
1- 2	7,400	15,500	0	0	40,550
2- 3	1,357	16,857	0	-667	39,893
3- 4	1,357	18,214	0	-666	39,217
4- 5	1,357	19,571	0	-667	38,550
5- 6	1,357	20,928	0	-350	38,200
6- 7	1,357	22,285	0	-350	37,850
7- 8	1,357	23,642	950	-350	38,450
8- 9	1,358	25,000	0	-350	38,100
9-10	0	25,000	-6,600	-350	31,150
10-11	0	25,000	-2,600	-350	28,200
11-12	0	25,000	-2,600	-350	25,250
12-13	0	25,000	-3,600	-350	21,300

$p_x(t, t+1)$ = troops of type x put ashore (+) or departed (-) during time t to t+1 (t measured in days).

$M(0)$ = German troops on hand prior to engagement,

$N_1(0)$ = British troops on hand prior to engagement.

$N_2(0)$ = Greek troops on hand prior to engagement,

$N(0)$ = Allied troops on hand prior to engagement.

Table I is a summary of assumptions (B) and (C) above showing the number of troops on hand initially, number put ashore each day, number departed each day, and cumulative total of troops initially on hand or put ashore less troops departed at the end of each day.

It is recognized that the numbers shown in table I are nothing better than an estimate of the troop landings and withdrawals, but it is felt that the estimate is reasonably derived from the data presented in reference (a) as summarized in assumptions (B) and (C) above. Without a thorough examination of some of the references cited in reference (a) it is not possible to verify the accuracy of table I. However it is felt that table I is sufficiently accurate to permit an order of magnitude estimate to be made of the average casualty producing rates of the German and Allied troops. The procedure used to determine these rates will now be described briefly.

3. The differential equations derived from assumption (A) are:

$$(1) \quad \frac{dM}{dt} = p_M(t) - A \cdot N(t) \qquad \frac{dN}{dt} = p_N(t) - B \cdot M(t)$$

where t is measured in days and:

- $p_x(t)$ is the rate troops are put into or removed from battle, by landings or evacuations.
- $M(t)$ is the number of German troops on hand at time t.
- $N(t)$ is the number of British and Greek (hereafter called Allied) troops on hand at time t.
- A is the average rate at which each Allied troop caused casualties to German troops.
- B is the average rate at which each German troop caused casualties to Allied troops.

The boundary conditions which must be satisfied are:

$$(2) \quad \begin{array}{ll} M(0) = 0 & M(13) = \begin{array}{l} 19{,}000 \text{ (German Estimate)} \\ 10{,}000 \text{ (British Estimate)} \end{array} \\ N(0) = 40{,}550 & N(13) = 0 \end{array}$$

The problem of solving the differential equation (1) for M(t) and N(t) in terms of unknown A and B and the specified values of M(0) and N(0) is not difficult, but the subsequent task of finding the values of A and B which yield the correct values of M(13) and N(13) is not simple computationally.

Consequently, the differential equations (1) have been replaced by difference equations which closely approximate them. The difference equations and boundary conditions may then be manipulated to yield the following recursive relationships:

$$
\begin{aligned}
&M(0) = 0 \qquad\qquad\qquad N(0) = 40{,}550 \\
&M(t+1) = M(t) + P_M(t,\ t+1) - AN(t) \quad \text{for } t \geq 0 \\
(3)\quad &N(1) = N(0) + P_N(0,\ 1) - B/2\left[M(1) + M(0)\right] \\
&N(t+1) = N(t) + P_N(t,\ t+1) - BM(t) \quad \text{for } t \geq 1
\end{aligned}
$$

where $p_x(t,\ t+1)$ is, as before, the number of troops of type x added to or evacuated from the engagement.

Given any set of values for A and B, it is a simple task to compute M(t), N(t) for any positive integral values of t and in particular M(13) and N(13). A straightforward successive approximation technique may then be used to choose successive values of A and B which bring the computed values of M(13) and N(13) closer to the desired values quoted in equation (2) for either the British or German estimate.

Initial estimates of A and B were obtained as follows: The total number of man-days available ashore prior to casualties was computed for each side by the expression:

$$\sum_{i=0}^{12} \sum_{t=0}^{i} P_M(t,\ t+1) = 257{,}957 \text{ German man-days available ashore prior to casualty.}$$

$$\sum_{i=0}^{12} \sum_{t=0}^{i} P_N(t,\ t+1) = 457{,}250 \text{ Allied man-days available ashore prior to casualty.}$$

The approximate number of man-days lost due to casualties was computed by assuming roughly constant loss rate per day yielding a total number of man-days lost equal to:

(13/2)x 6,000 = 39,000 German man-days lost (German estimate)

(13/2)x 15,000 = 97,500 German man-days lost (British estimate)

(13/2)x21,300 = 138,450 Allied man-days lost

Subtracting the number of man-days lost from the man-days available ashore prior to casualties yielded the approximate number of man-days spent in combat for each side. These results are:

218,597 German man-days in combat (German estimate)
160,097 German man-days in combat (British estimate)
318,800 Allied man-days in combat

Dividing the number of casualties incurred on one side by the number of man-days spent in combat by the other side yields the approximate casualty producing rates:

$A = \frac{6,000}{318,800} = .019$ German casualties per Allied man-day in combat (German estimate)

$A = \frac{15,000}{318,800} = .047$ German casualties per Allied man-day in combat (British estimate)

$B = \frac{21,300}{218,597} = .098$ Allied casualties per German man-day in combat (German estimate)

$B = \frac{21,300}{160,097} = .133$ Allied casualties per German man-day in combat (British estimate)

Rounding off the above values and choosing a pair of values of A and B of .04 and .15 respectively (in conformance with the British estimate of 15,000 German casualties), total casualties to both sides were computed with the aid of equation (3). These did not come out 15,000 German and 21,300 Allied casualties as required, to conform with the German estimate, but rather about 14,000 and 20,500. Multiplying A by 15,000/14,000 yielded the next approximation or A' about .043.

Although B appeared too small it was left unchanged to isolate the effect of the correction of A to A'. Subsequent approximations showed .15 slightly too low a value for B.

Continuing this process, after five successive approximations, values of A and B of .042 and .155, respectively, yielding 15,088 German casualties and 21,188 British casualties, were obtained. These values were adjudged satisfactory to fit the British estimate of German casualties.

Similarly, starting with a rounded off pair of values of .02 and .10 for A and B, respectively, to conform with the German estimate of German casualties, after five successive approximations values of A and B were obtained of .0162 and .104, respectively, yielding 5,978 German casualties and 21,303 British casualties. The computed numbers of troops on hand at the end of each day for each of these sets of values of A and B are shown in tables II and III.

Comparison of table I with tables II or III will yield the cumulative casualties (and hence daily casualties) inflicted on either side in conformity with either the German or the British estimate of the number of German casualties.

The procedure denoted above might have been somewhat simplified, had the results of reference (d) been available when this analysis was originally performed.

TABLE II

TROOPS ON HAND AT TIME t (IN CONFORMITY WITH GERMAN ESTIMATE OF GERMAN CASUALTIES)
(A = .0162, B = .104)

Time t (days)	German troops on hand M(t)	Allied troops on hand N(t)	Relative Effectiveness $A/B\left[N(t)/M(t)\right]^2$
0	0	40,550	∞
1	7,441	40,163	4.5
2	14,190	39,388	1.2
3	14,908	37,256	.97
4	15,660	35,029	.8
5	16,449	32,734	.6
6	17,276	30,675	.5
7	18,138	28,516	.4
8	19,034	27,232	.3
9	19,952	24,903	.2
10	19,549	15,878	.1
11	19,291	10,894	.05
12	19,118	5,937	.02
13	19,022	-3	$.04 \times 10^{-8}$

TABLE III

TROOPS ON HAND AT TIME t (IN CONFORMITY WITH BRITISH ESTIMATES OF GERMAN CASUALTIES)
(A = .042, B = .155)

Time t (days)	German troops on hand M(t)	Allied troops on hand N(t)	Relative Effectiveness $A/B\left[N(t)/M(t)\right]^2$
0	0	40,550	∞
1	6,397	40,054	10.6
2	12,115	39,062	2.8
3	11,831	36,527	2.6
4	11,654	34,017	2.3
5	11,582	31,544	2.0
6	11,614	29,399	1.7
7	11,736	27,249	1.5
8	11,949	26,030	1.3
9	12,214	23,828	1.0
10	11,213	14,985	.5
11	10,584	10,297	.3
12	10,152	5,706	.09
13	9,912	182	$.9 \times 10^{-4}$

4. An examination of tables I, II, and III makes it possible to form a judgment concerning the relative merits of the German and British estimates of the number of German casualties. In a combat situation where assumption (A) holds, the relative effectiveness of the forces at any time is:

$$(A/B)\left[N(t)/M(t)\right]^2$$

This is based on the assumptions that there are no further additions to or withdrawals from the combat area (other than casualties) after time t and that all forces put ashore can retain their average effectiveness as long as they do not become casualties, and is a result of Lanchester's "square law" which applies to such situations (reference (c)) and which may be written:

$$A\left[(N(t))^2 - (N(t+t'))^2\right] = B\left[(M(t))^2 - (M(t+t'))^2\right]$$

With this equation it is easy to show that if $(A/B)\left[N(t)/M(t)\right]^2$ is greater than one, the N side will win the engagement; if it is less than one, the M side will win the engagement; if it equals one, the battle will be a draw.

An examination of table III shows that the relative effectiveness $(A/B)\left[N(t)/M(t)\right]^2$ has the value of 10.6 at t = 1 and does not decrease to the value of 1 until t = 9. This means that until t = 9, the Allied forces on hand and not casualties were superior to the German forces on hand and not casualties. Consequently, if the Allies had been able to leave their troops on Crete, fighting as effectively as before, without further withdrawals after the ninth day, the battle might have been a draw, or even possibly an Allied victory as the casualties computed from tables I and III for the Allies are probably too low on the last day, (undoubtedly many prisoners were taken on the last day) and probably too high earlier, so that the Allies probably had more effective troops at hand and the Germans less at t = 9 if the British estimate of German casualties is valid. Since the Germans put no more reinforcements on shore after the ninth day, this suggests that the issue was indeed in doubt until the end of the ninth day.

On the other hand, an examination of table II shows that the relative effectiveness has the value 4.5 at t = 1 and decreases to the value of 1.0 at t = 3. Then since the Germans introduced an additional 9,500 troops subsequent to t = 3 and the Allies withdrew about 18,600 troops after t = 3, if the German estimate of the number of German casualties is valid, it would appear that from t = 3 on the Germans had established their superiority and would win the battle.

The true situation probably lies between the two extremes described above. The author is personally inclined to feel that the German estimate of German casualties is closer to the actual facts than the British estimate. This feeling is based upon the observation that the newly drafted Greek troops, from about the end of the third day on, were deserting to hide with the population of Crete

as fast as their ammunition ran out, so that even if they had remained, their combat effectiveness would have decreased rapidly after $t = 3$. In addition, large numbers of British troops counted in these computations as effective were sick, wounded, and war-weary from earlier battles, hence not of great usefulness. Thus it appears doubtful that the Allies could have inflicted as many casualties on the Germans as the British estimated.

5. Inasmuch as the casualty-producing rate of the German troops includes the taking of large numbers of Allied prisoners, and it has not been possible to take into account the number of wounded Allied troops, it is also of interest to obtain the kill-producing rates of the troops on both sides. Of the 21,300 Allied casualties, 5,200 were killed. This makes the German rate of killing Allied troops 5,200/21,300 or .245 times the total German rate of inflicting casualties on Allied troops. The British estimate of German casualties asserts that 6,000 Germans were killed of 15,000 German casualties; the Germans provide no breakdown of their casualties. Using the proportionality factor thus obtained for either estimate, the Allied rate of killing German troops is 6,000/15,000 or .4 times the total Allied rate of inflicting casualties on German troops.

6. The casualty and kill-producing rates of both sides as well as the comparable rates for the battle of Iwo Jima (as derived in reference (b)) are listed in table IV.

Table IV shows clearly the difference between the battles of Iwo Jima and Crete. The U.S. won the battle of Iwo Jima by establishing huge numerical superiority (73,000 U.S. troops put ashore during the first six days to combat 21,500 Japanese troops on the island) with troops that were less effective man-for-man than the Japanese. This was undoubtedly due to the fact that the Japanese were well dug in to strong defensive positions, well supplied, and in all respects ready for the invasion, while of course the U.S. troops were forced to fight from exposed positions. The Germans won the battle of Crete without establishing numerical superiority in the early stages of the battle (or indeed until about the tenth day) because they were more effective man-for-man than the Allies. This was due to the fact that the German troops were fresh, highly trained and well organized, and exercised excellent judgment in their use of tactics which prevented the Allied strongpoints from maintaining contact with each other, while the Allied troops were (except for a small nucleus of the original garrison) sick, wounded, combat-weary, poorly equipped, some barely trained (the Greek two-week soldiers) and some unarmed (the Cyprian and Palestinian labor forces).

TABLE IV

CASUALTIES OR KILLS INFLICTED ON OPPOSING SIDE
PER MAN-DAY OF COMBAT

Battle	Victor's combat effectiveness		Loser's combat effectiveness	
Crete	.104 - .155	Allied casualties per German man-day of combat	.0162 - .042	German casualties per Allied man-day of combat
	.0255-.0380	Allies killed per German man-days of combat	.0065-.0168	Germans killed per Allied man-days of combat
Iwo Jima	.0088-.0106	Japanese killed per U.S. man-day of combat	.0544	U.S. casualties per Japanese man-day of combat
			.0113	U.S. troops killed per Japanese man-day of combat

(The smaller figures are believed to be more credible in the Battle of Crete as the British estimate of German casualties is believed to be too high. The larger figure for U.S. effectiveness in the Battle of Iwo Jima is believed to be more creditable as it was computed under the assumption that a U.S. troop became ineffective when killed, wounded, or missing in action.)

It is interesting to note that there is very little variation in the kill-producing rates of the various types of troops which participated in these two engagements. But a clear warning must be sounded to discourage the drawing of any conclusion to the effect that there is such a thing as an operational constant such as "opposing troops killed per man-day of combat" which can be applied automatically to any campaign. Such kill-producing rates are affected by a wide variety of conditions, some of which have been discussed here, others of which can be thought of readily.

References: (a) ORO/JHU: Unpublished ORO Analysis of the Airborne Invasion of Crete 1953

(b) A Verification of Lanchester's Law, J.H. Engel JORSA Vol. 2, No. 2, May 1954

(c) Methods of Operations Research, Morse and Kimball John Wiley and Sons, Inc. 1951

(d) More on the Fiske Warfare Model, J.H. Engel, Operations Research, Vol. II, No. 1 Jan-Feb 1963